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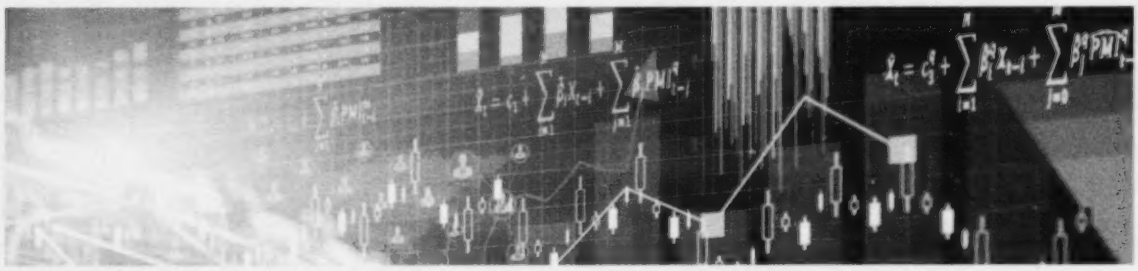
Banking Regulation and Market Making



by David A. Cimon and Corey Garriott

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Abstract

We present a model of market makers subject to recent banking regulations: liquidity and capital constraints in the style of Basel III and a position limit in the style of the Volcker Rule. Regulation causes market makers to reduce their intermediation by refusing principal positions. However, it can improve the bid-ask spread because it induces new market makers to enter. Since market makers intermediate less, asset prices exhibit a liquidity premium. Costs of regulation can be assessed by measuring principal positions and asset prices but not by measuring bid-ask spreads.

Bank topics: Financial markets; Market structure and pricing; Financial system regulation and policies
JEL codes: G14; G20; L10

Résumé

Nous présentons un modèle de teneurs de marché soumis aux règles de la réglementation bancaire récente : ratios de liquidité et exigences de fonds propres inspirés de Bâle III, limite de position inspirée de la règle Volcker. La réglementation amène les teneurs de marché à réduire leur activité d'intermédiation en refusant de prendre des positions de contrepartiste. Elle peut toutefois améliorer l'écart acheteur-vendeur, car elle induit l'entrée de nouveaux teneurs de marché. Comme les teneurs de marché ont réduit leur activité d'intermédiation, les prix des actifs affichent une prime de liquidité. Les coûts associés à la réglementation peuvent être évalués à partir des positions de contrepartiste et des prix d'actifs et non sur la base des écarts acheteur-vendeur.

Sujets : Marchés financiers; Structure de marché et fixation des prix; Réglementation et politiques relatives au système financier
Codes JEL : G14; G20; L10

Non-Technical Summary

After the 2007–2008 financial crisis, banking regulators in many countries coordinated a set of financial reforms for banks. The reforms, Basel III, are the third iteration of reforms coordinated by the Basel Committee on Banking Supervision (BCBS), a global forum for banking regulators. Basel III is designed to strengthen the financial position of banks by asking them to obtain funding in safer ways and to hold less risky kinds of assets. A related rule in the United States, the Volcker Rule of the 2012 Dodd–Frank Act, puts further constraints on US banks by restricting what and how much they can trade. Taken together, the new financial regulations are designed to increase financial stability, which is a potential benefit, but they may also create unintended costs for financial markets. Specifically, they may unduly constrain the ability of banks to buy and sell securities on demand for their clients, which is a necessary service.

A recent body of academic literature has attempted to measure the size of the costs imposed by Basel III and Volcker. If banks are less able to trade for their clients, it diminishes market liquidity. Liquidity is the cost charged by intermediaries for the service of transacting a security in volume and with immediacy. Academics have found that the impact of regulation on liquidity depends on how liquidity is measured. This paper contributes by giving a theoretical explanation accounting for the various results.

In our model, when regulations are imposed on the banks, they intermediate fewer client trades. However, the markup charged to clients is reduced. The reason they charge better prices is competition. The regulated banks leave some clients underserved, which creates the incentive for outside banks to start new market-making shops, and the market becomes more competitive. Although banks charge a lower markup, they do much less intermediation. As a consequence, asset prices become more sensitive to sudden demands to trade, as banks are less willing and able to act as intermediaries.

We conclude that there are indeed costs to regulation in financial markets and that they can be measured not in terms of markups but in terms of a reduced quantity of bank intermediation. The model provides some direction for future work. For accounting reasons, regulation in the style of Basel III should have a stronger impact when banks are purchasing from clients. If so, distinguishing liquidity when banks purchase securities from when banks sell securities may help distinguish the impacts of the new regulations, which are difficult to distinguish in the data.

"It is not clear whether there is or is not a problem."

—Janet Yellen, Governor, US Federal Reserve, on bond market liquidity

Authorities in more than 20 countries are implementing global reforms on banking. The reforms are designed to strengthen financial stability but may carry unintended consequences for securities markets. Critics argue the reforms have caused market makers to provide worse liquidity, particularly for corporate bonds.[1] Academics have examined the effects of regulation on corporate-bond liquidity but find mixed evidence. Trebbi and Xiao (2015) find no change in nine liquidity metrics; Bessembinder et al. (2016) find transaction-cost metrics improve, whereas dealer capital-commitment declines; and Dick-Nielsen and Rossi (2016) and Bao et al. (2016) find immediacy has become more costly.

In this paper, we construct a new model of market making in fixed income to understand the issue. Our model has two features new to the literature. First, we include financing decisions, which are important to understand liquidity in fixed income. Market makers issue debt and equity to finance their operations, and they use securities financing to manage their inventory. Second, we motivate market making using quantity competition. Market makers compete for separate buyer and seller markets on a Cournot basis, which creates a spread between the bid and ask.

Our model is the first to explain the apparently disparate findings from fixed-income data in a single framework. While the regulations do limit market making, the bid-ask spread is unchanged or even improved, as in Trebbi and Xiao (2015). However, the market is less able to handle sudden demand, as in Dick-Nielsen and Rossi (2016). While the cost of regulation does not appear in the bid-ask spread, we predict that it does appear in a reduced capacity

[1]See Duffie (2012) and "Top of Mind: A Look at Liquidity," Goldman Sachs, Global Macro Research, August 2015; "Global financial markets liquidity study," PwC, August 2015; "Addressing Market Liquidity: A Broader Perspective on Today's Bond Markets." BlackRock, Viewpoint, October 2016.

to bear inventory, as in Bessembinder et al. (2016). Market makers move to intermediate on an agency basis, again as in Bessembinder et al. (2016), and there is entry of outside market makers, as in Bao et al. (2016). In addition, we offer predictions to help assess the impact of the reforms. We predict that inventory premia or "price pressure" should increase and that the Basel III reforms should affect market makers' long positions but not their short positions.

Many of the results rely on the role of securities financing in bank market making and its accounting treatment. Fixed-income market makers finance purchases using repurchase agreements (repo) and procure securities for sale using reverse-repurchase agreements (reverse repo).[2] Repo is considered cash borrowing and is accounted as debt, while reverse repo is considered investment and is accounted as an asset.[3] As a consequence, constraints on debt or liabilities apply to repo but not reverse repo. Therefore, certain regulations affect only long positions. Though these regulations are intended to impact the banking side of the business, their impact on repo exposures directly impacts market making by banks as well.

In our results, we first analyze liquidity requirements. The Basel III liquidity requirements ask banks to carry saleable assets, intended to sufficiently cover liabilities at various terms.[4] In our model, we assume all repo liabilities have the same term, so we model liquidity requirements as a requirement to purchase a "high-quality liquid asset" (HQLA) in a percentage of repo. In the results, liquidity regulation adds the price of HQLA as a variable cost but only to long positions, creating a kink in the cost schedule. Market makers respond by avoiding small long positions, which are too small to merit paying the discontinuity in

[2]For an introduction to repo markets, see Garriott and Gray (2016) and Fontaine et al. (2016).

[3]According to generally accepted accounting principles (GAAP) and the International Financial Reporting Standards (IFRS).

[4]For an introduction to Basel III, see Chouinard and Paulin (2014).

cost. For larger long positions, the market makers pass part of the cost to clients by lowering the asset price. The cost leads to market-maker exit, and here the bid-ask spread worsens.

Second, we analyze capital requirements. The Basel III capital requirements ask banks to fund using a minimum of safe capital (equity and subordinated debt) in some proportion of assets. In our model, we assume market makers intermediate a single asset, so we model capital requirements as an upper bound on debt financing. In the results, constrained market makers respond by intermediating on an agency basis—they match investor buys directly with investor sells because taking on further inventory is impossible. When market makers do not absorb liquidity demand, the asset price must move to equilibrate investor buying and selling. However, the bid-ask spread improves. The regulation makes space on the balance sheet scarce, so firms enter to provide a good in demand. Since more firms compete to do less principal intermediation, the bid-ask spread tightens.

Third, we analyze the Volcker Rule. Volcker asks bank market makers to satisfy metrics related to risk, turnover and position. We assume the market already satisfies the risk and turnover metrics, and we model Volcker as simply a position limit, which is thought to be its most binding aspect. When the position limit binds, the analysis of Volcker is the same as it is for capital requirements, as the limit on repo financing is already a position limit. The sole difference is that the position limit binds on both long and short positions, whereas capital requirements bind only on long positions.

Last, we study a "short-term" case. The result that bid-ask spreads improve depends on entry, which blunts the impact of regulation. To study a shorter term, we fix the number of firms and again apply the regulations. In this case the regulations have uniformly negative impacts on spreads and have worse impacts on prices. This case is consistent with a sudden demand for immediacy, as in Dick-Nielsen and Rossi (2016) and Bao et al. (2016).

We conclude that regulation does create liquidity costs for markets, but the costs are unlikely to appear in average market-liquidity metrics. Instead, the costs appear in new inventory premia in asset prices or in new costs to immediacy. The results point to ways to distinguish the regulations. Costs deriving from Basel III are one-sided as they apply to long positions, whereas costs deriving from Volcker are two-sided. Last, the model makes predictions about long-term market structure changes. The market should experience the entry of outside liquidity suppliers, and banks should move to intermediate on an agency basis. Note that this paper is about the costs of regulation and not the benefits. Basel III and Volcker were designed to increase financial stability. The topic of this paper is the unintended consequences on costs, which are less well understood.

A. Literature

The model is a contribution to theory in market microstructure. The topic of balance-sheet regulation on market makers demands a framework that can blend market microstructure with corporate finance. It is not straightforward to embed corporate finance in the workhorse models of microstructure because they are models of agency friction, particularly private information, which creates tractability problems. Private information is less important in bond markets because most information on bond valuation is public (Fleming and Remolona, 1999). Instead of agency friction, we create illiquidity through imperfect competition in separated buyer and seller markets. The approach is simple but not common in the microstructure literature; similar models such as Liu and Wang (2016) or Vives (2011) retain information frictions and a single market. As Cournot is linear-quadratic, the setup is flexible enough to be deployed for other topics that demand a tractable way to handle corporate finance in broker-dealer markets.

The model assumes securities financing is frictionless, and we do not otherwise explore the securities-financing market or its role. Huh and Infante (2016) do study the importance of repo transactions to dealer intermediation in some detail. Bottazzi et al. (2012) and Foley-Fisher et al. (2015) also study the role of securities financing, and regulators are aware it is the practical basis of bond market making (Fontaine et al., 2016). Our contribution relative to this work is to focus on regulation. We impose three regulations on a bond market and show that the intuition that they hurt equilibrium market liquidity is muted or reversed once we allow for endogenous entry of market makers.

Evidence from corporate bond markets corroborates the model's predictions on liquidity. Liquidity metrics since the reforms show no evidence of deterioration in liquidity levels or risks (Trebbi and Xiao, 2015; Adrian et al., 2015) or even a mild improvement (Bessembinder et al., 2016). Liquidity metrics observed around shocks tell a different story. Dick-Nielsen and Rossi (2016) and Bao et al. (2016) exploit events that create a sudden need for immediate trading and find immediacy has become significantly more expensive. These results can be explained by our model in our "short-term" results, which hold the number of market makers constant.

Evidence from foreign exchange (FX) prices corroborates the model's predictions on asset prices. Pinnington and Shamloo (2016) and Du et al. (2016) find large and persistent deviations in covered interest parity in FX markets, which they attribute to a binding leverage ratio. Our results support their attribution because the deviations from parity are mostly negative in sign, which would result if borrowing is more difficult than lending, an outcome of a binding Basel III capital constraint. More work should be done to measure the reaction of fixed-income prices to regulation.

Finally, evidence on dealer behaviour corroborates the model's predictions on allocations. Bessembinder et al. (2016) and Bao et al. (2016) find deterioration in dealers' balance-sheet commitment and an increase in the percentage of trades conducted on an agency basis. Dealers who are less affected by regulation step in to provide liquidity. These results are consistent with our model, which predicts that market makers shift toward an agency basis of trading and that outside market makers step in.

This paper is structured as follows. In Section 1, we set up a model of capital structure and market making. In Section 2, we present its baseline equilibrium. In Section 3, we subject market makers to a Basel-like liquidity requirement. In Section 4, we subject market makers to a Basel-like leverage requirement. In Section 5, we study the interaction between the two requirements. Last, in Section 6 we present an alternative formulation of the model with a position limit as in the Volcker Rule. Proofs and figures are in Appendix A.

I. Model

We present a model of Cournot-competitive market makers. This model has three periods: the financing period $t = 0$, the trading period $t = 1$ and the liquidation period $t = 2$. Financial institutions must pay an upfront cost in order to become market makers. Market makers who choose to enter the market act as Cournot competitors. In order to finance their costs, market makers must issue either debt or equity and pay a market rate of return.

A. Assets

Traded Asset There exists an asset in unlimited supply with a known value v_0. This asset can be obtained only by market makers through the repo and reverse repo markets. Following trading, the asset payoff is realized.

Outside of the market makers, there exists aggregate demand to both buy and sell the asset. Market makers are able to buy the asset against an inverse demand curve of:

$$P_B = v_0 - l_B + B. \tag{1}$$

Similarly, market makers are able to sell the asset against an inverse demand curve of:

$$P_S = v_0 + l_S - S. \tag{2}$$

The values B and S represent the total quantities that maker makers choose to buy from and sell to the market, respectively. The values l_B and l_S are parameters representing the level of demand on each side of the market.

High-Quality Liquid Asset (HQLA)

An HQLA exists in unlimited quantity. This asset can be obtained frictionlessly, in unlimited supply, by market makers at a normalized value of 1. If a market maker purchases a supply H of this asset, the market maker earns a profit of $H(1 + r_F)$ following the trading period.

Sale and Repurchase Agreements (Repos) and Reverse Repos

During trading, market makers have access to frictionless markets for sale and repurchase agreements (repos) and reverse repo markets. If clients wish to sell more than they wish to buy, market makers require an additional supply of cash. They may conduct a repo transaction, paying a rate r_R, to borrow cash on a secured basis. As occurs in accounting practice, a repo transaction adds additional leverage to a market maker's balance sheet. If market demand of the traded asset exceeds market supply, market makers require an additional supply of the asset. They may conduct a reverse repo transaction, earning a rate

r_R, to acquire this asset. Unlike a repo, a reverse repo does not add additional leverage to a market maker's balance sheet.

A feature of repo is that, unlike the other assets in this model, financial institutions in this model do not consider them when calculating their return on assets. They must, however, consider them when calculating their liquidity and leverage ratios.

B. Financial Institutions

There exist an infinite number of risk-neutral financial institutions, indexed i. Each financial institution can choose to pay an exogenous cost c in order to become a market maker and solve the market maker's problem. This cost must be financed in $t = 0$ through either debt D_0 or equity E_0. The total rate of return required on the firm's assets is exogenously set as $1 + r_A$. We assume that all financial institutions will choose an equal debt/equity ratio in equilibrium.

By assumption, if a market maker requires additional funds during trading, it must do so by issuing debt. In the context of this model, this occurs when a financial institution wishes to acquire a quantity of HQLA. Unless this additional debt is a short-term repo, the market maker must also earn the return $1 + r_A$ on this additional debt. This additional debt is designated D_1.

Each financial institution solves three problems: (1) the entry problem in $t = 0$; (2) the corporate financing decision in $t = 0$; and (3) the market maker's problem in $t = 1$.

The firm's entry problem is to choose whether to pay $D_0 + E_0 = c$ in order to become a market maker. Each firm does so if it can earn a return $(1 + r_A)$ on its assets.

The firm's corporate financing problem is to choose initial values D_0 and E_0 in order to maximize the final value of the firm $D_2 + E_2$, given the results of market making.

The market maker's problem is to maximize his profits, from market making, given each of the demands l_B, l_S. Each firm that participates in market making realizes a profit of:

$$\pi_i = (v_0 - P_B)b_i + (P_S - v_0)s_i - r_R v_0(b_i - s_i) - (1 + r_A)c. \tag{3}$$

C. Market Structure and Timing

At the beginning of the model at $t = 0$, all financial institutions observe the market demand to buy l_B and sell l_S, and choose whether to pay the cost c to become a market maker. The firms then choose to finance this cost c through some combination of debt D_0 and equity E_0.

After the market making firms have completed their financing decisions, $t = 1$ begins. The values l_S and l_B realize and become publicly available. Each firm selects a quantity b_i to buy from the market and s_i to sell to the market, giving aggregate quantities of $B = \sum_i b_i$ and $S = \sum_i s_i$. The markets clear at prices P_B and P_S.

Finally, at $t = 2$, market makers realize the value of their remaining inventory. The firms liquidate and distribute their profits $D_2 + E_2$ to their financiers.

II. Baseline Equilibrium

In this section, we present the baseline equilibrium. In order to acquire cash to take a long position when market making ($b_i > s_i$), market makers engage in a repo transaction at a cost r_R. To acquire inventory to take a short position when market making ($b_i < s_i$), market makers engage in a reverse-repo transaction, earning a return r_R.

An equilibrium in the baseline model consists of: (i) a solution to each market maker's profit maximization problem for each liquidity realization (l_B, l_S); (ii) a solution to each

financial institution's corporate finance problem; and (iii) an equilibrium number of entrants N^* such that: If N^* financial institutions enter, each earns a return of at least $1 + r_A$ on its assets. Were $N^* + 1$ institutions to enter, each would earn less than $1 + r_A$ on its assets.

Theorem 1 (Existence of a Baseline Equilibrium):

(i) Given a number of entrants N, there exist unique liquidity supplies b_i and s_i for each market maker i, such that each market maker solves his optimization problem.

(ii) Given (i), there exist debt and equity values D_0 and E_0, such that each financial institution maximizes its firm value.

(iii) Given (i) and (ii), there exists a unique equilibrium number of entrants N^* such that each financial institution earns a return of at least $1 + r_A$ on its assets. Instead, were $N^* + 1$ financial institution to enter, each would earn less than $1 + r_A$ on its assets.

A. Market-Making Decision

Definition 1: *The cost of accessing the repo market, absent further constraints, is denoted as $\gamma = 2r_B v_0$.*

Given a number of entrants N and realizations l_B and l_S, each market maker chooses b_i and s_i to maximize:

$$\pi_i = (-\frac{\gamma}{2} + l_B - \sum_i b_i)b_i + (\frac{\gamma}{2} + l_S - \sum_i s_i)s_i - (1 + r_A)c. \tag{4}$$

The result of this unconstrained optimization problem yields the standard symmetric Cournot results, with liquidity supplies of:

$$b_i = \frac{l_B - \frac{\gamma}{2}}{N + 1} \tag{5}$$

$$s_i = \frac{\frac{\gamma}{2} + l_S}{N+1}. \tag{6}$$

These liquidity supply results and their accompanying prices can be seen in Figures 1 and 2.

B. Corporate Financing Decision

The baseline equilibrium for the corporate financing decision follows from the standard Modigliani-Miller results. The financial institution must finance $c = D_0 + E_0$ in order to become a market maker. Throughout trading, the market maker has no requirement to issue any long-term debt, but holds short-term debt equal to its repo position $D_1 = v_0(b_i - s_i)$, which it clears following trading.

For a financial institution trying to maximize its final value $V = D_2 + E_2$, any initial combination of debt and equity is equally optimal without additional financial market constraints.

C. Market Maker Entry

The profit of each market maker in each state is given by:

$$\pi = \frac{\left(l_B - \frac{\gamma}{2}\right)^2 + \left(l_S + \frac{\gamma}{2}\right)^2}{(N+1)^2} - (1 + r_A)c. \tag{7}$$

Given the expected profit, financial institutions choose whether to become market makers. Institutions will continue to enter as long as $\pi_i > 0$. Since π is decreasing in N, there is a single equilibrium number of entrants N^* such that:

$$N^* \leq \sqrt{\frac{(l_B - \frac{\gamma}{2})^2 + (l_S + \frac{\gamma}{2})^2}{(1 + r_A)c}} - 1 < N^* + 1. \tag{8}$$

III. Liquidity Ratio Constraint

In this section, we present a liquidity ratio requirement for financial institutions. We model this as being similar in nature to the Liquidity Coverage Ratio (LCR) in the Basel III agreement. This requirement obliges institutions to hold an HQLA (modelled as a risk-free asset) if they take on short-term repo liabilities. They must purchase this asset using cash acquired from their primary funding.

Assumption 1: *Financial institutions that wish to engage in a repo transaction must hold an HQLA. For a repo transaction of size $(b_i - s_i)$, they must hold $H_i = \alpha(b_i - s_i)v_0$. This HQLA earns a return of $H_i(1+r_F)$ and costs the firm $H_i(1+r_A)$ to be raised from its funding.*

Theorem 2 (Existence of a Liquidity Ratio Constraint Equilibrium):

Under a liquidity ratio constraint, there exists a unique equilibrium that is consistent with the properties of the equilibrium in the baseline model.

A. Market-Making Decision

Given a number of entrants N and realizations l_B and l_S, each market maker chooses b_i and s_i to maximize:

$$\pi_i = (-r_R v_0 + l_B - \sum_i b_i)b_i + (r_R v_0 + l_S - \sum_i s_i)s_i + (1+r_F)H_i - (1+r_A)(c+H_i), \quad (9)$$

where the amount of HQLA is equal to $H_i = \alpha(b_i - s_i)v_0$ if the market maker wishes to access the repo market ($b_i > s_i$). If the market maker optimally chooses to sell more than he buys, the result is unchanged from the baseline model. This occurs in equilibrium if $l_S - l_B > -\gamma$. If the market maker optimally chooses to buy more than he sells, he must hold HQLA. In

equilibrium, this costs him an additional $\alpha(r_A - r_F)v_0$ per unit. Alternatively, the market maker may choose to hold zero inventory ($b_i = s_i$) and avoid paying the costs associated with the repo market.

Definition 2: *The cost of accessing the repo market in the presence of a liquidity constraint is denoted by* $\Gamma = 2v_0(r_R + \alpha(r_A - r_F))$.

Given these costs, the market maker's liquidity supply functions are kinked, based on whether he chooses to access the repo market. In equilibrium, they are given by:

$$b_i = \begin{cases} \frac{l_B - \frac{\gamma}{2}}{N+1} & \text{if } l_S - l_B \geq -\gamma \\ \frac{l_S + l_B}{2(N+1)} & \text{if } -\gamma > l_S - l_B \geq -\Gamma \\ \frac{l_B - \frac{\Gamma}{2}}{N+1} & \text{if } l_S - l_B < -\Gamma \end{cases} \tag{10}$$

$$s_i = \begin{cases} \frac{\frac{\gamma}{2} + l_S}{N+1} & \text{if } l_S - l_B \geq -\gamma \\ \frac{l_S + l_B}{2(N+1)} & \text{if } -\gamma > l_S - l_B \geq -\Gamma \\ \frac{l_S + \frac{\Gamma}{2}}{N+1} & \text{if } l_S - l_B < -\Gamma. \end{cases} \tag{11}$$

These kinked liquidity supply functions lead directly to kinks in the market maker's profit function. In the first segment, they optimally access the repo market and hold HQLA when $l_S - l_B < -\Gamma$. In the second segment, they optimally hold zero inventory, when $-\gamma > l_S - l_B \geq -\Gamma$. The final segment, where they behave as in the baseline model, occurs when $l_S - l_B \geq -\gamma$. When the supply functions become kinked, prices also drop in response, leading to asset pricing effects. The difference in supply results and their accompanying prices and spreads can be seen in Figures 1, 2 and 3.

Proposition 1 (Relevance of the Liquidity Ratio Constraint):

In equilibrium, the liquidity ratio affects the market maker only if l_B is such that $l_B - l_S \geq \gamma$. Otherwise, the liquidity ratio has no effect.

Proposition 1 details the circumstances under which the liquidity ratio affects market maker decision-making. This corresponds to the point where the market maker wishes to buy more than he wishes to sell, and must consider purchases of HQLA. In that sense, the liquidity ratio affects market makers under normal market circumstances. If a market maker expects that he will need to access the short-term repo market in order to respond to market selling pressure, he also needs to consider holding HQLA in order to cover his impending cash outflows.

Once the market maker begins holding HQLA in order to cover his repo positions, he undergoes a level shift downward on his willingness to provide liquidity at the bid. On the other hand, because of this cost of providing liquidity at the bid, market makers become more willing to provide liquidity at the ask in order to balance their total position. Thus, the two sides of the market have opposite effects from the introduction of a liquidity ratio constraint. This results in the first testable implication of our model.

Testable Implication 1: *If the market maker is restricted by a liquidity ratio, liquidity will be restricted when market selling pressure exceeds market buying pressure. That is to say, market makers will provide more liquidity when selling assets than when buying assets.*

Definition 3: *The "short-run analysis" is defined by fixing the number of market makers, N. The "long-run analysis" is defined by allowing the optimal entry of new market makers.*

A further interpretation of the equilibrium comes from fixing the number of entrants N, a case which we refer to as the "short-run analysis." This short-run analysis serves as a measure of the costs of immediacy because the industrial organization cannot adapt to buying and

selling interest. For a fixed number of market makers, an increase in selling pressure from the market, I_B, results in an increase in the quantity bought by market makers, with no other effect. This is because once market makers begin holding HQLA, they are willing to intermediate by taking on principal positions. Further, the total number of market makers in the market does not affect their willingness to hold HQLA. While the spread will be wider from the lack of competitive entry, there is no additional immediacy cost from the liquidity constraint. The price effects of this short-run analysis are shown in Figure 4.

B. Corporate Financing Decision

As in the baseline, the financial institution must finance $c = D_0 + E_0$ in order to become a market maker. However, now the market maker must issue additional debt during trading in order to purchase HQLA to cover any repo position. His debt during trading is now a combination of this debt to cover the repo position and the debt to purchase HQLA $D_1 = v_0(1 + \alpha)(b_i - s_i)$. As before, it clears following trading. For a financial institution trying to maximize its final value ($V = D_2 + E_2$), any initial combination of debt and equity remains optimal.

Proposition 2 (Capital Structure Irrelevance):

In both the baseline model and the model with a liquidity ratio constraint, Modigliani-Miller capital structure irrelevance holds.

As the market maker's profitability is not affected by his corporate financing decisions, Modigliani-Miller capital structure irrelevance holds in both the baseline model and the model with a liquidity ratio constraint. It should be noted that Proposition 2 holds despite the assumption that market makers must finance with debt during trading, rather than

financing with additional equity. Thus, absent further frictions, this assumption does not affect the Modigliani-Miller results.

C. *Market Maker Entry*

The profit of the market maker depends on each segment of his market-making strategy. These segments follow the same kinks as those in the liquidity supply function.

$$\pi_i = \begin{cases} \frac{\left(l_B - \frac{\gamma}{2}\right)^2 + \left(l_S + \frac{\gamma}{2}\right)^2}{(N+1)^2} - (1 + r_A)c & \text{if } l_S - l_B \geq -\gamma \\ \frac{(l_B + l_S)^2}{2(N+1)^2} - (1 + r_A)c & \text{if } -\gamma > l_S - l_B \geq -\Gamma \\ \frac{\left(l_B - \frac{\Gamma}{2}\right)^2 + \left(l_S + \frac{\Gamma}{2}\right)^2}{(N+1)^2} - (1 + r_A)c & \text{if } l_S - l_B < -\Gamma. \end{cases} \tag{12}$$

We denote these equations by π_1, π_2 and π_3, respectively. The equilibrium number of entrants, given the underlying parameter venues, is then obtained by solving each of the preceding equations.

Proposition 3 (Market Power Under a Liquidity Ratio Constraint):

If a market maker is affected by the liquidity constraint and must hold HQLA, he earns lower profit for a given number of entrants N than in the baseline model. In equilibrium, if market makers are affected by the funding constraint, there will be fewer entrants N^* than in the baseline model.

When market makers are affected by a liquidity constraint, they earn fewer profits for any given number of entrants N. As the cost of becoming a market maker is unaffected, fewer market makers enter in equilibrium. Each market maker who does choose to enter then commands more market power in the presence of a liquidity ratio constraint. Further, as the market makers compete in a Cournot fashion, having fewer entrants results in a lower total supply of the asset during trading, and lower consumer surplus.

IV. Leverage Ratio Constraint

In this section we relax Assumption 1 and subject the market maker to a leverage ratio. Specifically, the ratio of the total value of the institution's debt (D_0 from the initial financing decision and D_1 from the repo market) to its total value must not increase above some exogenous number.

Assumption 2: *Financial institutions are subject to a leverage ratio constraint, represented by β. The maximum amount of debt an institution may hold is $\beta \geq \frac{D_0+D_1}{D_0+D_1+E_0}$.*

The equilibrium in this section is unlike that in the baseline model. While for given a number of entrants N the equilibrium supply of assets for a given number of entrants is unique, the equilibrium number of entrants itself may not be unique. Under certain parametrizations, there exist two equilibria for the number of entrants, denoted by N^* and N_B. N^* is the equilibrium in the case where the leverage constraint does not bind, while N_B is the number in the case where it does. Depending on the parametrization of the model, the equilibrium may be unique and be defined by only N^* or N_B. Alternatively, there may exist two equilibria, one in which the leverage constraint binds, and one in which it does not. The parametrizations under which this occurs are explored later.

Theorem 3 (Existence of a Leverage Ratio Constraint Equilibrium):

(i) Given a number of entrants N, there exist unique liquidity supplies b_i and s_i for each market maker i, such that each market maker solves his optimization problem.

(ii) Given (i), there exist debt and equity values D_0 and E_0, such that each financial institution maximizes its firm value.

(iii) Given (i) and (ii), there exists either a single unique equilibrium, or two equilibria for the number of entrants, denoted by N^* and N_B. N^* is the equilibrium number of entrants such that the leverage ratio does not bind and each financial institution earns a return of $1 + r_A$ on its assets. N_B is the equilibrium number of entrants such that the leverage ratio does bind and each financial institution earns a return of $1 + r_A$ on its assets. In each case, were an additional firm to enter, each firm would earn less than $1 + r_A$ on its assets.

A. Market-Making Decision

When considering the market-making decision, the market maker faces two scenarios. Either the market maker may be able to market make without being bound by the leverage ratio, or the market maker is affected by the leverage ratio. The latter case occurs only when the market maker has to take on additional leverage during the market-making phase ($b_i > s_i$). The conditions under which these scenarios occur are detailed in the corporate financing decision below.

Definition 4: *The maximum additional debt a firm can add during market making, in the presence of a leverage constraint, is denoted by $D_1 = \psi$.*

Consider a market maker with the ability to take on additional debt of $v_0(b_i - s_i) \leq \psi$. With the baseline equilibrium supply, the leverage ratio does not bind when:

$$l_B - l_S \leq \frac{(N+1)\psi}{v_0} + \gamma. \tag{13}$$

Consider a market maker who, using the baseline model, wishes to choose asset supplies that violate the leverage ratio constraint such that $v_0(b_i - s_i) > \psi$. The market maker finds himself unable to take on further debt, and is constrained in his ability to access the repo market. The market maker can sell additional units to buy more units without taking on further debt. If the market maker wishes to buy, he must sell a number of units equal to:

$$s_i = b_i - \frac{\psi}{v_0}. \tag{14}$$

This market maker's problem is then to maximize a new profit function, given by:

$$\pi_i = (-\frac{\gamma}{2} + l_B - \sum_i b_i)b_i + (\frac{\gamma}{2} + l_S - \sum_i b_i + \sum_i \frac{\psi}{v_0})(b_i - \frac{\psi}{v_0}) - (1 + r_A)c. \tag{15}$$

As in the case of the liquidity ratio constraint, this results in kinked liquidity supply curves. If, using the baseline equilibrium supply and given l_S and l_B, the market maker does not reach ψ, the leverage constraint does not bind. It is optimal for him to continue using the baseline equilibrium supplies. If the leverage ratio does bind, his liquidity supply curves are the result of the above profit function. In equilibrium, the liquidity supply decisions are then given by:

$$b_i = \begin{cases} \frac{l_B - \frac{\gamma}{2}}{N+1} & \text{if } l_B - l_S \leq \frac{(N+1)\psi}{v_0} + \gamma \\ \frac{l_B + l_S}{2(N+1)} + \frac{\psi}{2v_0} & \text{if } l_B - l_S > \frac{(N+1)\psi}{v_0} + \gamma. \end{cases} \tag{16}$$

$$s_i = \begin{cases} \frac{\frac{\gamma}{2}+l_S}{N+1} & \text{if } l_B - l_S \leq \frac{(N+1)\psi}{v_0} + \gamma \\ \frac{l_B+l_S}{2(N+1)} - \frac{\psi}{2v_0} & \text{if } l_B - l_S > \frac{(N+1)\psi}{v_0} + \gamma. \end{cases} \tag{17}$$

Whereas the liquidity constraint causes a level shift downward in the market maker's willingness to buy assets, the leverage constraint creates a slope change. The market maker becomes increasingly less willing to supply liquidity at the bid, and begins supplying additional liquidity at the ask in order to stay within his leverage ratio. In the extreme case, market makers may sell the asset at below value in order to generate the additional ability to buy more assets from the market. This remains optimal, as long as the market selling pressure is sufficiently high such that the market maker is able to buy from the market at a lower price than he is able to sell. These changes from the baseline can be seen in Figures 1 and 2.

The leverage ratio has different implications for short-run analysis than the liquidity constraint. If the number of market makers N is fixed, and there is an increase in selling pressure, l_B, market makers will be constrained by the leverage ratio sooner. In the long run, as selling pressure increases, the number of market makers would also increase, creating competition and leaving more room on each market maker's balance sheet. Further, once the leverage ratio becomes binding, an increase in selling pressure increases both buying and selling by market makers. Thus, without market maker entry, not only do spreads increase but the effects of the leverage ratio are exacerbated. This increase in the cost of immediacy is similar to the results in both Dick-Nielsen and Rossi (2016) and Bao et al. (2016). The amplified price effects of this short-run analysis are shown in Figure 4.

B. Corporate Financing Decision

The leverage constraint imposes a limit on the financial institution, both in its initial financing decision and in accessing the repo market. As in the baseline, the financial institution must finance $c = D_0 + E_0$ in order to become a market maker and take on on additional debt when accessing the repo market, with $D_1 = v_0(b_i - s_i)$.

Proposition 4 (Loss of Capital Structure Irrelevance):

In the model with a leverage ratio constraint, Modigliani-Miller capital structure irrelevance does not hold. Financial institutions are able to increase their value by choosing an initial level of debt and equity such that: $D_0 \leq \frac{\beta E_0 - (1-\beta)v_0(b_i - s_i)}{1-\beta}$.

Unlike the baseline model, if the leverage constraint binds, the market maker is not able to engage in further repo transactions. The market maker's optimal solution is to take a debt position that is sufficiently low that the leverage constraint does not bind. This is a result of the assumption that the market maker must use debt during trading in order to generate additional cash. While this assumption does not remove capital structure irrelevance in the baseline case, or the case with a liquidity ratio constraint, the leverage ratio constraint does incentivize particular capital structures.

In the corner case, if the optimal value of debt were to be negative, the optimal solution is one in which the market maker takes an unlevered position $E_0 = c$. In this case, the leverage ratio binds and the maximum repo transaction is of size:

$$\psi = \frac{\beta c}{1-\beta}. \tag{18}$$

Unlike the liquidity ratio constraint, the leverage ratio constraint is much more limited in its application. For a bank that optimally chooses a capital structure low in long-term debt, this ratio binds only when the market maker faces very heavy selling pressure.

Proposition 5 (Relevance of the Leverage Ratio):

In equilibrium, the leverage ratio affects the market maker only if $l_B - l_S > \frac{(N+1)\phi}{v_0} + \gamma$. *Otherwise, the leverage ratio has no effect.*

By comparing the results of Propositions 1 and 5, we show that the leverage ratio affects the market maker much less frequently than the liquidity ratio. In fact, the necessary level of sell pressure for the leverage ratio to affect the market is strictly greater than that of the liquidity ratio, regardless of the actual policy levels of these two ratios.

Testable Implication 2: *If the market maker is restricted by a leverage ratio constraint, liquidity will be restricted only during periods of heavy selling pressure. During normal trading, liquidity will not be restricted on either side of the market.*

The differences between Propositions 1 and 5 also imply that the effects from a leverage constraint are empirically differentiable from the effects of a liquidity ratio constraint. Whereas the liquidity ratio constraint affects the market makers any time they take a position that forces them to access the repo market, the leverage ratio constraint binds only under limited circumstances. If a market maker is performing optimally, his capital structure should be sufficiently equity-based that only a large amount of short-term debt can bind the leverage ratio. Mathematically, this is past the point where the liquidity ratio would already have had an effect. Thus, the effects of a liquidity ratio should be seen during much more regular conditions than the leverage ratio.

C. *Market Maker Entry*

In equilibrium, given the kink of the liquidity supply functions, the market maker's profit function is also kinked. For a given number of entrants N, it is given by:

$$\pi_i = \begin{cases} \frac{1}{(N+1)^2}((l_B - \frac{\gamma}{2})^2 + (l_S + \frac{\gamma}{2})^2) - (1 + r_A)c & \text{if } l_B - l_S \leq \frac{(N+1)\psi}{v_0} + \gamma \\ \frac{(l_B+l_S)^2}{2(N+1)^2} + \frac{\psi(l_b - l_s - \gamma)}{2v_0} - \frac{N\psi^2}{2v_0^2} - (1 + r_A)c & \text{if } l_B - l_S > \frac{(N+1)\psi}{v_0} + \gamma. \end{cases} \tag{19}$$

When the market maker's leverage constraint does not bind, the number of entrants in equilibrium N^* is identical to that in the baseline model. When it does bind, the equilibrium number of entrants N_B is such that if N_B^* solves the equation:

$$(1 + r_A)c = \frac{(l_B + l_S)^2}{2(N_B^* + 1)^2} + \frac{\psi(l_b - l_s - \gamma)}{2v_0} - \frac{N_B^*\psi^2}{2v_0^2}, \tag{20}$$

then $N_B \leq N_B^* < N_B + 1$. Unlike the baseline model, where the equilibrium number of entrants is unique, the leverage ratio constraint introduces the possibility of two equilibria. For an underlying parameter set, there exist two equilibria, one with N^* entrants in which the leverage ratio does not bind, and one with N_B entrants, in which the leverage ratio does bind if:

$$N_B < \frac{v_0}{\psi}(l_B - l_S - \gamma) - 1 \leq N^*. \tag{21}$$

V. Interaction Between Liquidity Ratio and Leverage Ratio Constraints

In this section, we subject the market maker to both a leverage ratio and a liquidity ratio requirement by imposing Assumptions 1 and 2 simultaneously. These two regulatory requirements compound the effects described in previous sections. First, if the market maker wishes to access the repo market when market making, he must hold a supply of HQLA in order to cover his repo position. Second, the market maker must take on additional leverage in order to purchase the HQLA, adding to the leverage created when accessing the repo market.

The equilibrium in this section is consistent with the leverage-ratio-only equilibrium. As in the previous section, the equilibrium supply of assets for a given number of entrants is unique. However, the possibility of multiple equilibria for the number of entrants exists.

Theorem 4 (Existence of a Combined Constraint Equilibrium):

Under both a liquidity ratio and leverage ratio constraint, there exists either a single unique equilibrium, or two equilibria, consistent with the properties of the equilibria in the case of a leverage ratio constraint.

A. Market-Making Decision

The equilibrium market-making decision when dealing with both liquidity and leverage ratio constraints is a combination of the two previous sections. When the leverage ratio does not hold, the market maker behaves as in the liquidity ratio case. When the leverage ratio does hold, the market maker performs as in the leverage ratio case.

Definition 5: *The maximum additional debt a firm can add during market making when purchasing the tradeable asset, in the presence of both liquidity and leverage constraints, is denoted* $D_1 = \Psi$.

Consider a market maker who is about to take on a total additional leverage of $v_0(b_i - s_i) \leq \Psi$. The unconstrained decision is identical to that in the baseline model, while the constrained decision is identical to the one presented in Section IV. In equilibrium, liquidity supply is given by:

$$b_i = \begin{cases} \frac{l_B - \frac{\gamma}{2}}{N+1} & \text{if } l_S - l_B \geq -\gamma \\ \frac{l_S + l_B}{2(N+1)} & \text{if } -\gamma > l_S - l_B \geq -\Gamma \\ \frac{l_B - \frac{\Gamma}{2}}{N+1} & \text{if } -\frac{(N+1)\psi}{v_0} - \Gamma \leq l_S - l_B < -\Gamma \\ \frac{l_B + l_S}{2(N+1)} + \frac{\Psi}{2v_0} & \text{if } l_S - l_B < -\frac{(N+1)\Psi}{v_0} - \Gamma \end{cases} \tag{22}$$

$$s_i = \begin{cases} \frac{\frac{\gamma}{2} + l_S}{N+1} & \text{if } l_S - l_B \geq -\gamma \\ \frac{l_S + l_B}{2(N+1)} & \text{if } -\gamma > l_S - l_B \geq -\Gamma \\ \frac{l_S + \frac{\Gamma}{2}}{N+1} & \text{if } -\frac{(N+1)\psi}{v_0} - \Gamma \leq l_S - l_B < -\Gamma \\ \frac{l_B + l_S}{2(N+1)} - \frac{\Psi}{2v_0} & \text{if } l_S - l_B < -\frac{(N+1)\Psi}{v_0} - \Gamma. \end{cases} \tag{23}$$

B. Corporate Financing Decision

Unlike the case with only the leverage ratio, the additional debt must now cover the repo position and the HQLA purchase from the liquidity ratio. This total additional debt is $D_1 = v_0(1+\alpha)(b_i - s_i)$. The corner case in which the firm reaches its leverage constraint

worsens from the leverage constraint alone, and the maximum size of the repo transaction is:

$$\Psi = \frac{\beta c}{(1-\beta)(1+\alpha)}. \tag{24}$$

Proposition 6 (Maximum Liquidity with Liquidity and Leverage Constraints):

(i) All else being equal, the maximum additional debt a market maker can take on, Ψ, is lower under both leverage and liquidity ratio constraints than under a leverage ratio constraint alone.

(ii) The leverage ratio has an effect on the market maker if $l_B - l_S \geq \frac{(N+1)\Psi}{v_0} + \Gamma$. The necessary size of this shock may by greater or less than the necessary size in the case of a leverage ratio constraint alone.

The first portion of Proposition 6 is a comparison between the corner solutions to the leverage ratio given by Equations 18 and 24. When a market maker must hold the HQLA to cover a repo position, the additional debt used to purchase the HQLA compounds the debt of the repo position. Thus, the market maker adds both stocks of debt to his leverage ratio, and the leverage ratio binds for smaller total positions.

However, compared with Proposition 5, the level of sell pressure required to create an effect from the leverage ratio is ambiguous. The requirement to hold HQLA lowers the total amount of repo market debt that the market maker can take on; however, it also lowers the market maker's willingness to access the repo market, for any given level of sell pressure. This second effect lowers his total willingness to take on debt for any given demand shock.

C. *Market Maker Entry*

The profit of the market maker follows the kinks in the supply function. This profit function is:

$$\pi_i = \begin{cases} \frac{\left(l_B-\frac{\gamma}{2}\right)^2+\left(l_S+\frac{\gamma}{2}\right)^2}{(N+1)^2} - (1+r_A)c & \text{if } l_S - l_B \geq -\gamma \\ \frac{(l_B+l_S)^2}{2(N+1)^2} - (1+r_A)c & \text{if } -\gamma > l_S - l_B \geq -\Gamma \\ \frac{\left(l_B-\frac{\Gamma}{2}\right)^2+\left(l_S+\frac{\Gamma}{2}\right)^2}{(N+1)^2} - (1+r_A)c & \text{if } -\frac{(N+1)\psi}{v_0} - \Gamma \leq l_S - l_B < -\Gamma \\ \frac{(l_B+l_S)^2}{2(N+1)^2} + \frac{\Psi(l_b-l_s-\Gamma)}{2v_0} - \frac{N\Psi^2}{2v_0^2} - (1+r_A)c & \text{if } l_S - l_B < -\frac{(N+1)\Psi}{v_0} - \Gamma. \end{cases} \tag{25}$$

In the case where the leverage ratio does not bind, the number of entrants is identical to that of the model with the liquidity ratio constraint only. When the leverage constraint does bind, the equilibrium number of entrants N_B is such that if N_B^* solves:

$$(1+r_A)c = \frac{(l_B+l_S)^2}{2(N_B^*+1)^2} + \frac{\Psi(l_b-l_s-\Gamma)}{2v_0} - \frac{N_B^*\Psi^2}{2v_0^2}, \tag{26}$$

then $N_B \leq N_B^* < N_B + 1$. Like the case of the leverage constraint alone, there exists the possibility of two equilibria. For an underlying parameter set, there exist two equilibria, one with N^* entrants in which the leverage ratio does not bind, and one with N_B entrants in which the leverage ratio does bind if:

$$N_B < \frac{v_0}{\Psi}(l_B - l_S - \Gamma) - 1 \leq N^*. \tag{27}$$

VI. Extension: Position Limits and the Volcker Rule

In this section we consider an alternative policy to the leverage and liquidity ratio constraints, modelled based on one portion of the Volcker Rule. The Volcker Rule does not ascribe a firm position limit; rather, it institutes a ban on proprietary trading by financial institutions with access to US Federal backstops. These institutions must supply an array of metrics, including their internal position limits, in order to prove compliance with this ban. We model market-maker behaviour based on this position limit and show that it is similar to the leverage ratio. However, unlike the leverage ratio, it applies in cases where there is heavy market buy pressure as well as heavy market sell pressure. Formally, we relax Assumptions 1 and 2 and introduce Assumption 3.

Assumption 3: *Market makers have an exogenous position limit* Δ*, such that their net position must be below this limit following trading (*$|b_i - s_i| \leq \Delta$*).*

An equilibrium in this section is similar to that in the case of the leverage ratio alone. The liquidity supply parameters are unique for any given number of entrants N, but the number of entrants N itself may have multiple equilibria. Below, we present only the market maker's decision portion of this equilibrium, in order to derive testable implementations of a position limit versus a leverage ratio.

A. Market-Making Decision

The market maker's decision under a position limit is similar to that under a leverage ratio. If the market maker's optimal liquidity supply, using the baseline results, is such that the position limit does not bind ($b_i - s_i| < \Delta$), then there is no effect. In equilibrium, this occurs when:

$$|l_B - l_S - \gamma| < (N+1)\Delta. \tag{28}$$

In the case where the market is experiencing heavy sell pressure ($l_B - l_S - \gamma > (N+1)\Delta$), the market maker wishes to violate his position limit and it binds with equality. The market maker sets $s_i = b_i - \Delta$ and maximizes his profit function. Alternatively, in the case where the market is experiencing heavy buy pressure ($l_S + \gamma - l_B > (N+1)\Delta$), the market maker sets $b_i = s_i - \Delta$ and maximizes his profit function. In equilibrium, his liquidity supply functions are:

$$b_i = \begin{cases} \frac{l_B + l_S}{2(N+1)} - \frac{\Delta}{2} & \text{if } l_S + \gamma - l_B > (N+1)\Delta \\ \frac{l_B - \frac{\gamma}{2}}{N+1} & \text{if } |l_B - l_S - \gamma| < (N+1)\Delta \\ \frac{l_B + l_S}{2(N+1)} + \frac{\Delta}{2} & \text{if } l_B - l_S - \gamma > (N+1)\Delta \end{cases} \tag{29}$$

$$s_i = \begin{cases} \frac{l_B + l_S}{2(N+1)} + \frac{\Delta}{2} & \text{if } l_S + \gamma - l_B > (N+1)\Delta \\ \frac{\frac{\gamma}{2} + l_S}{N+1} & \text{if } |l_B - l_S - \gamma| < (N+1)\Delta \\ \frac{l_B + l_S}{2(N+1)} - \frac{\Delta}{2} & \text{if } l_B - l_S - \gamma > (N+1)\Delta. \end{cases} \tag{30}$$

Testable Implication 3: *If the market maker is bound by the Volcker Rule, liquidity supply should be restricted during periods of heavy buy pressure and heavy sell pressure. If, instead, the market maker is bound by the leverage ratio, liquidity supply should be restricted only during heavy sell pressure.*

This additional kink in the supply functions offers differentiation from the effects of a leverage ratio alone. While the leverage ratio limits liquidity supply during periods of heavy selling pressure, the Volcker Rule should limit liquidity during periods of heavy buy-

ing pressure as well. Empirically speaking, this limits liquidity during market swings in either directly, whereas the leverage ratio should limit liquidity only during heavy market downturns.

As with the leverage ratio, the effects of the position limit are highly dependent on the number of entrants N. In the short run, for a fixed number of entrants, the effects of the position limit are exacerbated. Market makers reach their position limit sooner, which affects both sides of the market, raising the cost of immediacy.

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A Appendix

A. Baseline Equilibrium

Proof of Theorem 1

Parts i, ii and iii

The proof of the baseline equilibrium is consistent with a standard problem of Cournot competition with endogenous entry. The market maker has a profit function denoted:

$$\pi_i = (-r_R v_0 + l_B - \sum_i b_i)b_i + (r_R v_0 + l_S - \sum_i s_i)s_i - (1 + r_A)c. \tag{A.1}$$

The problem is solved in the standard way, first by taking the first-order conditions with respect to b_i and s_i for all i. Second, the symmetry conditions of $b_i = b_j$ and $s_i = s_j$ are imposed for all N entrants, giving equilibrium liquidity supplies of

$$b_i = \frac{l_B - r_R v_0}{N + 1} \tag{A.2}$$

$$s_i = \frac{r_R v_0 + l_S}{N + 1}. \tag{A.3}$$

The prices at which the market maker buys and sells to the market are given by:

$$P_B = v_0 - \frac{N\frac{\gamma}{2} + l_B}{N + 1} \tag{A.4}$$

$$P_S = v_0 - \frac{N\frac{\gamma}{2} - l_S}{N + 1}. \tag{A.5}$$

The equilibrium liquidity supplies and prices can then be inserted into the profit functions. Through algebraic manipulation, the equilibrium profit is then given by:

$$\pi_i = \frac{1}{(N+1)^2}\left((l_B - r_R v_0)^2 + (l_S + r_R v_0)^2\right) - (1 + r_A)c. \tag{A.6}$$

This profit function is independent of the market maker's debt and equity choice, which therefore have no effect on the firm. The optimal combination of debt and equity is any $D_0, E_0 \geq 0$ such that $D_0 + E_0 = c$. In equilibrium, more firms will continue to enter until they earn zero profit. Setting the profit equation equal to 0, and solving for N, gives an equilibrium number of entrants equal to:

$$\dot{N} = \sqrt{\frac{(l_B - r_R v_0)^2 + (l_S + r_R v_0)^2}{(1 + r_A)c}} - 1. \tag{A.7}$$

As the number of firms is discrete, N^* is the integer value directly below the preceding equation, such that $N^* \leq \dot{N} < N^* + 1$. As with the rest of the baseline equilibrium, the equilibrium number of entrants is consistent with the general properties of a Cournot equilibrium with endogenous entry.

B. Liquidity Ratio

Proof of Theorem 2

Part i

This proof is in three subparts: (1) a market maker who accesses the reverse repo market by selling more units than he buys ($s_i > b_i$); (2) a market maker who holds zero net inventory ($s_i = b_i$); and (3) a market maker who accesses the repo market and must hold HQLA ($s_i < b_i$).

First, for a market maker who accesses the reverse repo market, the equilibrium supply, demand and prices are equal to those in the proof of Theorem 1, Part i. This strategy is

viable only when the market maker sells more units than he buys ($s_i > b_i$), or else he must pay the costs for HQLA. By substituting the equilibrium supply functions from the baseline model, it can be shown that the market maker's strategy results in $s_i > b_i$ when:

$$l_S - l_B \geq -2r_R v_0. \tag{A.8}$$

In this case, the market maker's profit function is identical to the baseline, denoted:

$$\pi_1 = \frac{1}{(N+1)^2}\left((l_B - r_R v_0)^2 + (l_S + r_R v_0)^2\right) - (1 + r_A)c. \tag{A.9}$$

Second, a market maker may wish to hold a positive inventory, but because of the cost of holding HQLA, holds a zero inventory instead. To hold a zero inventory, a market maker sets $s_i = b_i$ and maximizes his initial profit function. This results in equilibrium supplies of:

$$b_i = s_i = \frac{l_S + l_B}{2(N+1)}. \tag{A.10}$$

Given this equilibrium supply, the market prices are:

$$P_B = v_0 - \frac{(N+2)l_B - Nl_S}{2(N+1)} \tag{A.11}$$

$$P_S = v_0 + \frac{(N+2)l_S - Nl_B}{2(N+1)}. \tag{A.12}$$

Substitution into the profit function yields a profit of:

$$\pi_2 = \frac{(l_B + l_S)^2}{2(N+1)^2} - (1 + r_A)c. \tag{A.13}$$

This profit function is viable for any parameter set. However, by evaluating $\pi_2 < \pi_1$, algebraic manipulation shows that π_1 is preferred by market makers when: $l_S - l_B \geq -2r_R v_0$. Thus, the space spanned by these profit functions is mutually exclusive.

Finally, a market maker may wish to access the repo market and hold HQLA. In doing so, he maximizes his profit function while adding HQLA holds equal to $b_i - si$. These holdings pay a return of r_F and cost a rate of r_A. By taking the first derivatives of this profit function, equilibrium supplies are shown to be:

$$b_i = \frac{l_B - v_0(r_R + \alpha(r_A - r_F))}{N+1} \tag{A.14}$$

$$s_i = \frac{l_S + v_0(r_R + \alpha(r_A - r_F))}{N+1}. \tag{A.15}$$

Given the equilibrium supplies, the optimal prices are given by:

$$P_B = v_0 - \frac{Nv_0(r_R + \alpha(r_A - r_F)) + l_B}{N+1} \tag{A.16}$$

$$P_S = v_0 - \frac{Nv_0(r_R + \alpha(r_A - r_F)) - l_S}{N+1}. \tag{A.17}$$

Substituting these into the profit functions gives a total profit of:

$$\pi_3 = \frac{\left(l_B - \frac{\Gamma}{2}\right)^2 + \left(l_S + \frac{\Gamma}{2}\right)^2}{(N+1)^2} - (1 + r_A)c. \tag{A.18}$$

As the market maker holds HQLA only when he wishes to buy more than he sells, this strategy is not viable for any parameter set where the market maker wishes to supply $s_i > b_i$. In the baseline case, this occurs when $l_S - l_B \geq -2r_R v_0$, and thus the spaces spanned by π_3 and π_1 are mutually exclusive.

By evaluating $\pi_3 \geq \pi_2$, it can be shown that holding HQLA is preferred to holding a zero net inventory when:

$$l_S - l_B < -2v_0(r_R + \alpha(r_A - r_F)). \tag{A.19}$$

Thus, in equilibrium, the spaces over which these two functions are optimal are mutually exclusive.

As all three possible profit functions are optimal over mutually exclusive spaces that are collectively exhaustive over the parameter space, the market maker's choice of strategy exists and is unique for any given parameter set.

Part ii

As in the baseline model, none of the profit functions are dependent on the debt level of the market maker. Thus, the optimal combination of debt and equity is any $D_0, E_0 \geq 0$ such that $D_0 + E_0 = c$.

Part iii

Combining the profit functions from Theorem 2, Part i, through use of indicator functions, results in a total profit function given by:

$$\begin{aligned}\pi = &\mathbb{1}(l_S - l_B \geq -\gamma) \cdot \pi_1 \\ &+ \mathbb{1}(-\gamma > l_S - l_B \geq -\Gamma) \cdot \pi_2 \\ &+ \mathbb{1}(l_S - l_B < -\Gamma) \cdot \pi_3.\end{aligned} \tag{A.20}$$

The space spanned by the indicator functions is mutually exclusive and collectively exhaustive of the entire parameter space. Through algebraic manipulation the number of entrants N can by isolated and is given by:

$$\begin{aligned} N = \Bigg(\Big[& \mathbb{1}\left(l_S - l_B \geq -\gamma\right) \cdot \Pi_1 \\ & + \mathbb{1}\left(-\gamma > l_S - l_B \geq -\Gamma\right) \cdot \Pi_2 \\ & + \mathbb{1}\left(l_S - l_B < -\Gamma\right) \cdot \Pi_3 \Big] \cdot \frac{1}{c(1+r_A)} \Bigg)^{\frac{1}{2}} - 1, \end{aligned} \tag{A.21}$$

where $\Pi_i = (\pi_i + (1+r_A)c) \cdot (N+1)^2$. Each Π_i is a function of only the underlying parameter set and is greater than or equal to zero over the space defined by its respective indicator function. As in the baseline model, as the number of firms is discrete, N^* is the integer value directly below the solution to each of the above equations. Thus, N^* exists and is unique.

Proof of Proposition 1

As shown in Theorem 2, Part i, when the market maker sells more than he buys ($s_i > b_i$), he does not pay the costs of holding HQLA and his optimality decisions are unchanged from the baseline model. In the baseline equilibrium, $s_i > b_i$ when $l_S - l_B \geq -2r_B v_0$. Thus, substituting for γ, when $l_S - l_B \geq -\gamma$, the liquidity ratio has no effect on the market maker's decision.

Proof of Proposition 2

This follows from the Proof of Theorem 2, Part ii. As the firm's profit function is independent of its debt and equity structure, any combination of debt and equity results in the same value of the firm. Thus, the firm is unable to use debt or equity in order to increase its value, and the Modigliani-Miller theorem holds.

Proof of Proposition 3

A market maker holds HQLA or holds zero inventory only when $b_i \geq s_i$, which in equilibrium occurs when $l_S - l_B \geq -\gamma$. First, we evaluate the conditions $\pi_1 \geq \pi_2$ and $\pi_1 \geq \pi_3$. Algebraic manipulation shows that, were π_1 viable, these conditions would hold over the entire space where $l_S - l_B < -\gamma$ also holds. However, since π_1 is not viable, the market maker must choose either of the strategies resulting in π_2 or π_3. Thus, if the market maker is affected by the liquidity ratio, his profit is less than the profit he would earn in the baseline model given the same number of entrants.

Given that $\pi_1 \geq \pi_2$ and $\pi_1 \geq \pi_3$ over the space where $l_S - l_B < -\gamma$, then $\Pi_1 \geq \Pi_2$ and $\Pi_1 \geq \Pi_3$, where $\Pi_i = (\pi_i + (1 + r_A)c) \cdot (N+1)^2$. Thus, N^* from Theorem 2, Part iii is strictly lower than N^* from Theorem 1, Part iii, in any case where $l_S - l_B < -\gamma$.

C. Leverage Ratio

Proof of Theorem 3

Part i The proof of this section is in two parts: (1) when the leverage ratio does not bind, and (2) when the leverage ratio does bind.

First, if using the baseline equilibrium supply and given l_S and l_B, the market maker does not reach ψ, the leverage constraint does not bind. It is optimal for him to continue using the baseline equilibrium supplies. Substituting the baseline equilibrium supply into the leverage constraint, the leverage ratio does not bind when:

$$l_B - l_S \leq \frac{(N+1)\psi}{v_0} + \gamma. \tag{A.22}$$

As before, the market maker's profit function is identical to the baseline, denoted:

$$\pi_1 = \frac{1}{(N+1)^2}\left((l_B - r_R v_0)^2 + (l_S + r_R v_0)^2\right) - (1 + r_A)c. \tag{A.23}$$

Second, consider the market maker who, using the baseline model, wishes to choose asset supplies that violate Equation A.31 such that $r_0(b_i - s_i) > \psi$. If the market maker wishes to buy, he must sell a number of units equal to:

$$s_i = b_i - \frac{\psi}{v_0}. \tag{A.24}$$

Substitution into the market maker's initial profit function gives a new profit function of:

$$\pi_i = (-\frac{\gamma}{2} + l_B - \sum_i b_i)b_i + (\frac{\gamma}{2} + l_S - \sum_i b_i + \sum_i \frac{\psi}{v_0})(b_i - \frac{\psi}{v_0}) - (1 + r_A)c. \tag{A.25}$$

The equilibrium supply decisions are then given by:

$$b_i = \frac{l_B + l_S}{2(N+1)} + \frac{\psi}{2v_0} \tag{A.26}$$

$$s_i = \frac{l_B + l_S}{2(N+1)} - \frac{\psi}{2v_0}. \tag{A.27}$$

Given this equilibrium supply, the market prices are:

$$P_B = v_0 - \frac{(N+2)l_B - Nl_S}{2(N+1)} + \frac{N \cdot \psi}{2v_0} \tag{A.28}$$

$$P_S = v_0 + \frac{(N+2)l_S - Nl_B}{2(N+1)} + \frac{N \cdot \psi}{2v_0}. \tag{A.29}$$

Like the baseline model and the model with the liquidity ratio only, these decisions are unique for a given number of entrants N.

Part ii

Given debt D_0 and equity E_0 from the initial issuance, the maximum value of repo exposure is equal to:

$$\psi = v_0(b_i - s_i) = \frac{\beta E_0 - (1-\beta)D_0}{1-\beta}. \tag{A.30}$$

As shown in the Proof of Theorem 3, Part ii, the market maker's decisions in the market-making problem are affected by the value of the leverage constraint. Thus, the market maker optimally chooses an initial debt level that prevents the repo constraint from binding.

This is given by:

$$D_0 \leq \frac{\beta E_0 - (1-\beta)v_0(b_i - s_i)}{1-\beta}. \tag{A.31}$$

If there exist multiple values of $D_0, E_0 > 0$ such that $D_0 + E_0 = c$ and Equation A.31 holds, then any of these pairs are optimal and the leverage constraint does not bind.

If, however, there are no values $D_0, E_0 > 0$ such that $D_0 + E_0 = c$, then the leverage constraint does bind. Since the market maker's liquidity supply is increasing toward the baseline in ψ, his optimal decision is to maximize ψ. He does so by choosing an initial value of debt $D_0 = 0$ and financing entirely through equity $E_0 = c$. The maximum value of his new repo position, ψ, is then given by:

$$\psi = \frac{\beta c}{1-\beta}. \tag{A.32}$$

Thus, in equilibrium, the market maker's optimal solution to select any value $D_0 \geq 0$ such that Equation A.31 holds or $D_0 = 0$ as a corner solution.

Part iii

The profitability of the market maker depends on whether his leverage constraint binds. When the leverage constraint does not bind, his profit is identical to the baseline and liquidity ratio cases, denoted:

$$\pi_1 = \frac{1}{(N+1)^2}((l_B - \frac{\gamma}{2})^2 + (l_S + \frac{\gamma}{2})^2) - (1 + r_A)c. \tag{A.33}$$

When the leverage constraint does bind, substitution of the liquidity supply functions from Proof of Theorem 3 into the profit function gives:

$$\pi_B = \frac{(l_B + l_S)^2}{2(N+1)^2} + \frac{\psi(l_b - l_s - \gamma)}{2v_0} - \frac{N\psi^2}{2v_0^2} - (1 + r_A)c. \tag{A.34}$$

The combination of these two equations yields a total profit of:

$$\begin{aligned} \pi = & \mathbb{1}\left(l_B - l_S \leq \frac{(N+1)\psi}{v_0} + \gamma\right) \cdot \pi_1 \\ & + \mathbb{1}\left(l_B - l_S > \frac{(N+1)\psi}{v_0} + \gamma\right) \cdot \pi_B. \end{aligned} \tag{A.35}$$

While the equilibrium supply functions and prices are unique given the number of entrants, the number of entrants is not necessarily unique itself. When the market maker's leverage constraint does not bind, the number of entrants in equilibrium is identical to that in the baseline model. When it does bind, there exists N_B^*, which solves the equation:

$$(1+r_A)c = \frac{(l_B+l_S)^2}{2(N_B^*+1)^2} + \frac{\psi(l_b-l_s-\gamma)}{2v_0} - \frac{N_B^*\psi^2}{2v_0^2}. \tag{A.36}$$

As this equation is strictly decreasing in $N_B^* > 0$, the real value $N_B^* > 0$, which solves this equation, is unique. As when the leverage ratio does not bind, the number of entrants N_B is such that $N_B \leq N_B^* < N_B + 1$.

The number of entrants is unique if, for the values N^* and N_B above, the market maker's leverage constraint either binds when N is replaced by both N^* and N_B, or does not bind when N is replaced by both N^* and N_B. Consider the case where either N^* or N_B firms enter, and:

$$\begin{aligned} l_B - l_S &\leq \frac{(N^*+1)\psi}{v_0} + \gamma \\ &\leq \frac{(N_B+1)\psi}{v_0} + \gamma. \end{aligned} \tag{A.37}$$

If N^* firms enter, leverage constraint does not bind. Through the profit function in the baseline model each firm earns zero profit in expectation and, thus, this is an equilibrium. If N_B firms enter, the leverage constraint does not bind. Since $N_B < N^*$, substitution of the baseline liquidity supply into the profit function gives a profit greater than zero. Thus, more firms could have entered and this is not an equilibrium.

Alternatively, there is a unique equilibrium where the leverage constraint binds and N_B firms enter if:

$$l_B - l_S > \frac{(N^* + 1)\psi}{v_0} + \gamma \qquad (A.38)$$
$$> \frac{(N_B + 1)\psi}{v_0} + \gamma.$$

The uniqueness of this equilibrium is symmetric to the one above, with one alteration. If N^* firms enter, each earns less than zero profit and defaults, and if N_B firms enter, each earns zero profit. Thus, in this case, there is a unique number of entrants N_B.

Finally, there exist two equilibria if, given equilibrium entrants N^*, market makers earn zero profit and the leverage constraint does not bind, but given entrants N_B, market makers also earn zero profit and the leverage constraint does bind. This occurs, given values N^* and N_B defined above, if:

$$N_B < \frac{v_0}{\psi}(l_B - l_S - \gamma) - 1 \leq N^*. \qquad (A.39)$$

Thus, given the conditions above, there exists either: (1) a unique equilibrium number of entrants N^* or N_B, or (2) two equilibria number of entrants N^* or N_B.

Proof of Proposition 4

Proof follows from Proof of Theorem 3, Part ii. As the market maker is able to increase outputs by selecting a capital structure with $D_0 \leq \frac{\beta E_0 - (1-\beta)v_0 b_i}{1-\beta}$, then he is able to increase his firm's value through capital structure. This is driven by the assumption that the market maker is able to finance his subsequent market making costs only through additional debt (the repo market), rather than additional equity issuances.

Thus, Modigliani-Miller capital structure irrelevance does not hold under these assumptions.

Proof of Proposition 5

The leverage ratio affects the market maker if he wishes to take on more additional debt than his maximum allowable ψ, such that $\psi < v_0(b_i - s_i)$.

By substituting the equilibrium value of ψ from Proof of Theorem 3, Part ii, and the baseline liquidity supplies from Proof of Theorem 1, Part i, it can be shown that the market maker is affected by the leverage ratio only when:

$$l_B - l_S > \frac{(N+1)\psi}{v_0} + \gamma. \tag{A.40}$$

D. Interaction Between Liquidity Ratio and Leverage Ratio

Proof of Theorem 4

Part i

The proof of this section is in two parts: (1) when the leverage ratio does not bind, and (2) when the leverage ratio does bind.

First, when the leverage ratio constraint does not bind, the market maker maximizes a profit function identical to that in Proof of Theorem 2, Part i. The liquidity supply constraint binds when:

$$v_0(1+\alpha)(b_i - s_i) \geq \Psi. \tag{A.41}$$

Substituting the equilibrium values b_i, s_i from the case with the liquidity ratio alone, algebraic manipulation can show that the leverage constraint binds when:

$$l_S - l_B < -\frac{(N+1)\Psi}{v_0} - \Gamma. \tag{A.42}$$

In the case where the market maker holds HQLA and the leverage ratio binds, substituting the new position limit of $s_i = b_i - \frac{\Psi}{v_0}$ into the market maker's initial profit function gives a new profit function of:

$$\pi_i = (-\frac{\Gamma}{2} + l_B - \sum_i b_i)b_i + (\frac{\Gamma}{2} + l_S - \sum_i b_i + \sum_i \frac{\Psi}{v_0})(b_i - \frac{\Psi}{v_0}) - (1 + r_A)c. \tag{A.43}$$

Taking the first-order condition gives equilibrium liquidity supply values of:

$$b_i = \frac{l_B + l_S}{2(N+1)} + \frac{\Psi}{2v_0} \tag{A.44}$$

$$s_i = \frac{l_B + l_S}{2(N+1)} - \frac{\Psi}{2v_0}. \tag{A.45}$$

Thus, for any given value of N, there exist unique liquidity supply functions.

Part ii

Given debt D_0 and equity E_0 from the initial issuance, the maximum value of repo exposure is equal to:

$$v_0(1+\alpha)(b_i - s_i) = \Psi = \frac{\beta E_0 - (1-\beta)D_0}{1-\beta}. \tag{A.46}$$

As before, profit is decreasing when repo exposure is restricted. The market maker optimally chooses an initial debt level that prevents the repo constraint from binding. This is given by:

$$D_0 \leq \frac{\beta E_0 - (1-\beta)(1+\alpha)v_0(b_i - s_i)}{1-\beta}. \tag{A.47}$$

As a result of the additional leverage required to purchase HQLA, the corner case worsens and the maximum size of the repo transaction is:

$$\Psi = v_0(b_i - s_i) = \frac{\beta c}{(1-\beta)(1+\alpha)}. \tag{A.48}$$

Thus, as in the case with the leverage constraint alone, in equilibrium the market maker's optimal solution is to select any value $D_0 \geq 0$ such that Equation A.47 holds or $D_0 = 0$ as a corner solution.

Part iii

The profitability of the market maker depends on whether his leverage constraint binds. When the leverage constraint does not bind, his profit is identical to the liquidity ratio cases π_1, π_2 and π_3, as shown in Proof of Theorem 2, Part iii. When the leverage ratio does bind, substitution of the supply functions from Proof of Theorem 4, Part i results in a profit function of:

$$\pi_{B2} = \frac{(l_B + l_S)^2}{2(N+1)^2} + \frac{\Psi(l_b - l_s - \Gamma)}{2v_0} - \frac{N\Psi^2}{2v_0^2} - (1+r_A)c. \tag{A.49}$$

The combination of these equations yields a total profit of:

$$\begin{aligned}
\pi = {} & \mathbb{1}\left(l_S - l_B \geq -\gamma\right) \cdot \pi_1 \\
& + \mathbb{1}\left(-\gamma > l_S - l_B \geq -\Gamma\right) \cdot \pi_2 \\
& + \mathbb{1}\left(-\Gamma > l_S - l_B \geq -\frac{(N+1)\Psi}{v_0} - \Gamma\right) \cdot \pi_3 \\
& + \mathbb{1}\left(l_S - l_B < -\frac{(N+1)\Psi}{v_0} - \Gamma\right) \cdot \pi_{B2}.
\end{aligned} \tag{A.50}$$

In the case where the leverage ratio does not bind, the number of entrants N^* is identical to that of the model with the liquidity ratio constraint only. When the leverage constraint does bind, there exists $N_B^* > 0$, which solves:

$$(1 + r_A)c = \frac{(l_B + l_S)^2}{2(N_B^* + 1)^2} + \frac{\Psi(l_b - l_s - \Gamma)}{2v_0} - \frac{N_B^* \Psi^2}{2v_0^2}. \tag{A.51}$$

As before, the number of entrants N_B is such that $N_B \leq N_B^* < N_B + 1$. The uniqueness or non-uniqueness of the number of entrants follows from the Proof of Theorem 3, Part iii. The number of entrants is unique if, for a given l_B and l_S, there is only one incentive-compatible choice for market makers. Given the values N^* and N_B, there is a unique equilibrium where the leverage constraint does not bind and N^* firms enter if:

$$\begin{aligned} l_B - l_S &\leq \frac{(N^* + 1)\Psi}{v_0} + \Gamma \\ &\leq \frac{(N_B + 1)\Psi}{v_0} + \Gamma. \end{aligned} \tag{A.52}$$

Alternatively, there is a unique equilibrium where the leverage constraint binds and N_B firms enter if:

$$\begin{aligned} l_B - l_S &> \frac{(N^* + 1)\Psi}{v_0} + \Gamma \\ &> \frac{(N_B + 1)\Psi}{v_0} + \Gamma. \end{aligned} \tag{A.53}$$

There exist two equilibria if, given equilibrium entrants N^*, market makers earn zero profit and the leverage constraint does not bind, but given entrants N_B, market makers also earn

zero profit and the leverage constraint does bind. This occurs, given values N^* and N_B defined above, if:

$$N_B < \frac{v_0}{\Psi}(l_B - l_S - \Gamma) - 1 \leq N^*. \tag{A.54}$$

Proof of Proposition 6

Part i

A comparison between the maximum additional debt $\psi = \frac{\beta c}{1-\beta}$ under a leverage constraint alone, and $\Psi = \frac{\beta c}{(1-\beta)(1+\alpha)}$ under both a leverage and liquidity ratio constraint, shows that:

$$\Psi < \psi. \tag{A.55}$$

Intuitively, this follows from the fact that if the market maker wishes to take on additional leverage by accessing the repo market, he must take on further leverage in order to purchase HQLA and cover his repo position.

Part ii

As in Proof of Proposition 5, the leverage ratio affects the market maker if he wishes to take on additional debt $\Psi < v_0(b_i - s_i)$. When taking on debt, he has already accessed the repo market and therefore taken on HQLA purchases. Thus, substitution of the liquidity supply parameters from when the leverage constraint binds results in:

$$l_B - l_S \geq \frac{(N+1)\frac{\beta c}{(1-\beta)(1+\alpha)}}{v_0} + 2v_0(r_R + \alpha(r_A - r_F)). \tag{A.56}$$

Substituting for Ψ and Γ gives the final condition:

$$l_B - l_S \geq \frac{(N+1)\Psi}{v_0} + \Gamma. \tag{A.57}$$

Since $\Psi < \psi$, but $\Gamma > \gamma$, this condition may be greater than or less than the condition for the leverage ratio alone.

Figure 1
Long-Run Supply Given Fixed Buy Pressure

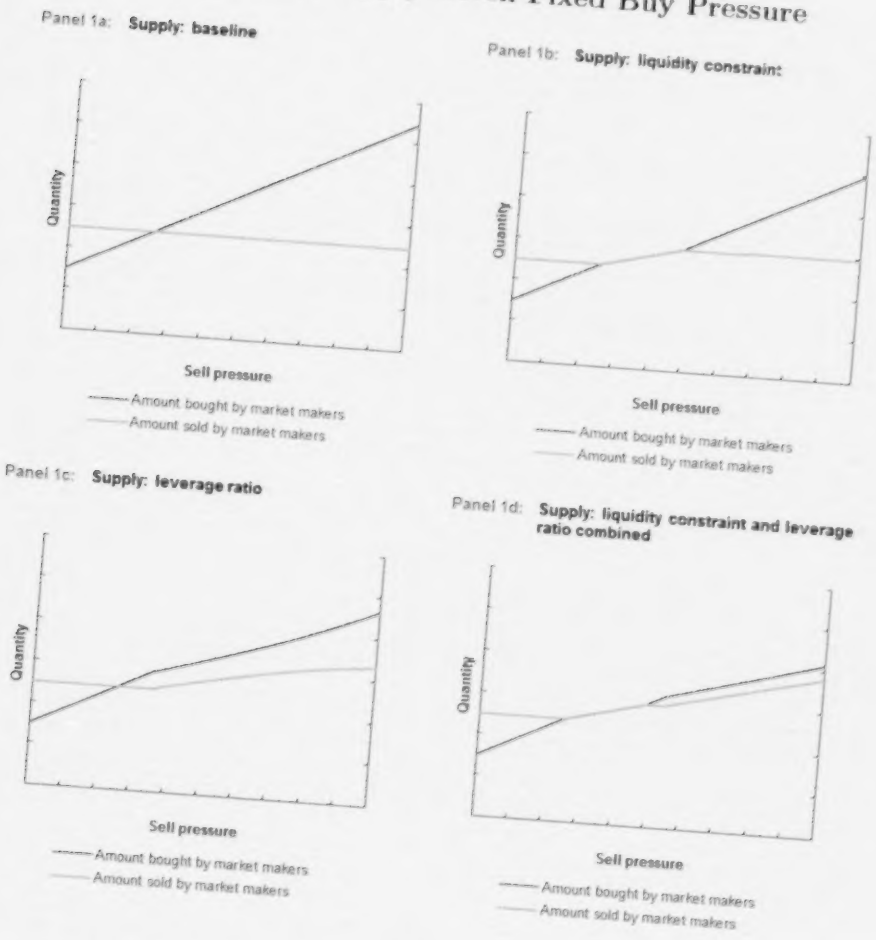

Figure 1 illustrates the quantity supplied by a market maker, given varying degrees of sell pressure l_B. The buying pressure l_S is assumed to be constant. In the baseline model, the quantity bought by the market maker increases linearly with buy pressure, while the quantity sold remains unaffected. The introduction of a liquidity constraint introduces kinks into the liquidity supply functions. The first kink represents the point where the market maker maintains zero net inventory in order to avoid holding HQLA, while the second kink represents the point where he begins holding HQLA. The introduction of a leverage ratio introduces a single kink in the supply function, at the point where the leverage ratio binds. At this point, the market maker begins selling a unit for every unit bought. Finally, when both constraints are imposed, the market maker's supply functions have a combination of the three kinks listed above.

Figure 2
Long-Run Prices Given Fixed Buy Pressure

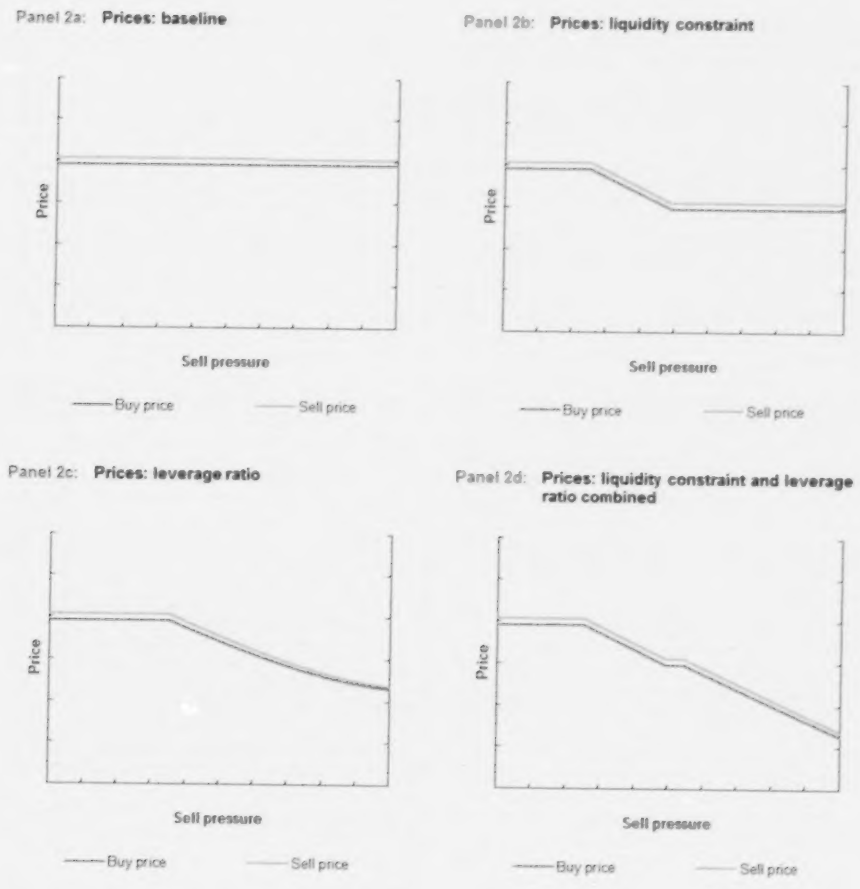

Figure 2 illustrates the prices at which market makers buy and sell the asset, given varying degrees of sell pressure l_B. The buying pressure l_S is assumed to be constant. In the baseline model, prices remain at approximately the value of the asset v_0, as more market makers enter when sell pressure increases. The introduction of a liquidity constraint introduces kinks into the price functions. When market makers must hold HQLA, their cost of the asset changes, making it cheaper for them to sell the asset than to buy it, shifting the price downward. The introduction of a leverage ratio also introduces a kink into the pricing function at the point where the leverage ratio binds. At this point, market makers are unable to buy as much as the baseline case and must sell in excess, reducing the price. Finally, when both constraints are imposed, the pricing functions have a combination of the above.

Figure 3
Long-Run Spreads Given Fixed Buy Pressure

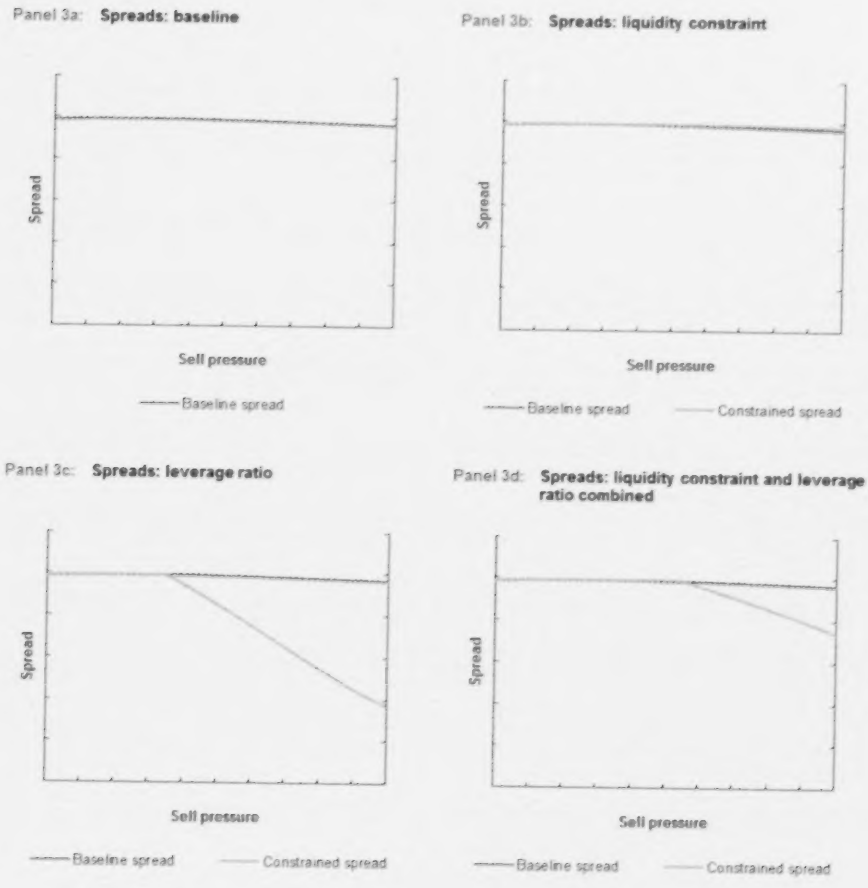

Figure 3 illustrates the bid-ask spread, given varying degrees of sell pressure l_B. The buying pressure l_S is assumed to be constant. In the baseline case, spreads slowly narrow as sell pressure increases. The entry of new market makers narrows spreads quicker than the increase in sell pressure increases them. With the introduction of a liquidity constraint, spreads narrow at a slightly slower rate, due to the cost of holding HQLA. The introduction of a leverage ratio introduces a kink in the spreads. When the leverage ratio binds, market makers are restricted in their ability to increase their amount bought and must increase their amount sold. As additional sell pressure is unserved by incumbents, new market makers are able to enter at a faster rate, decreasing spreads rapidly. Finally, when both constraints are imposed, spreads are a combination of the above cases.

Figure 4
Short-Run Prices Given Fixed Buy Pressure

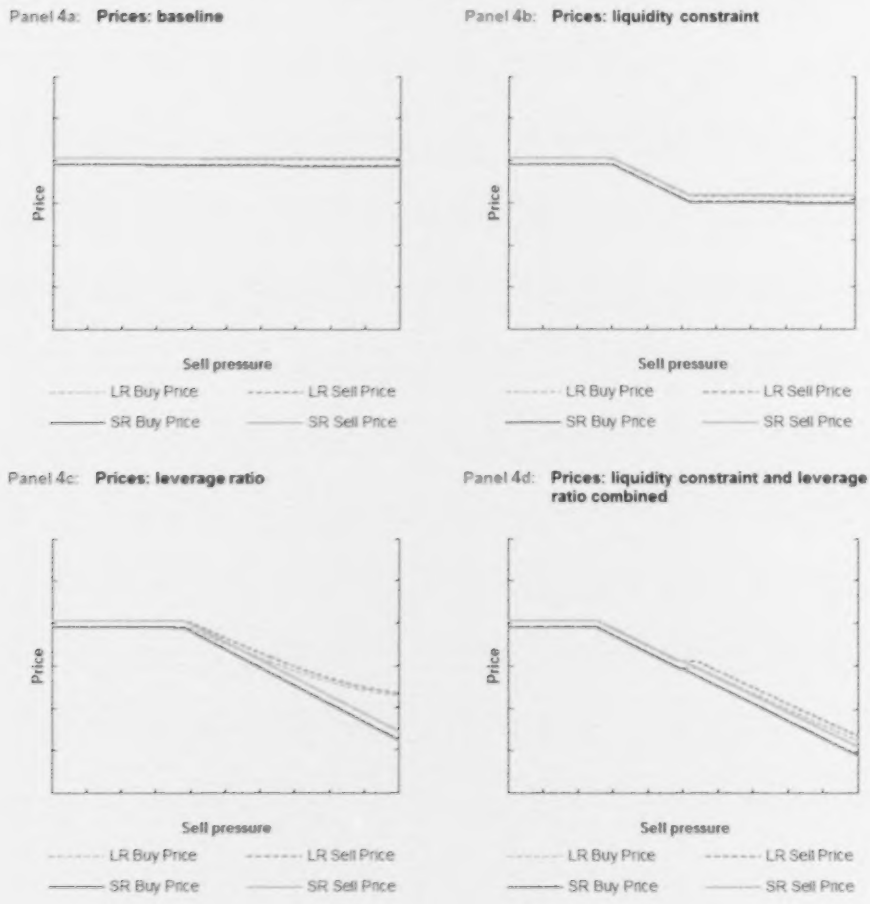

Figure 4 compares the prices in the short-run analysis with those in the long-run analysis. Unlike the long-run case, where entry is allowed, in the short-run analysis the number of entrants N is assumed to be constant. In the baseline model and the model with the liquidity constraint, spreads are wider when new entry is not allowed. With the leverage ratio, price pressure is exacerbated when new entry is not allowed, and represents an increase in the cost of immediacy. The case with both constraints imposed is a combination of the above.

Figure 5
Baseline Model: Flow Chart

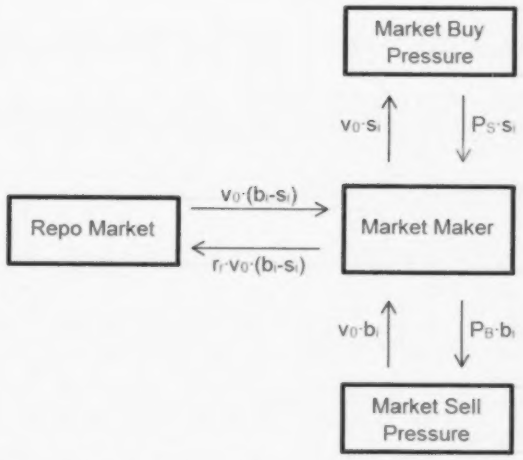

Figure 5 illustrates the baseline model. A group of N market makers provide liquidity by both buying and selling the asset as Cournot competitors. Market makers access the repo market in order to acquire either more assets or cash. If a market maker wishes to buy more than he sells, he requires cash to do so. He uses this excess inventory for collateral and acquires the cash through the repo market at cost r_R. If a market maker wishes to sell more than he buys, he requires additional assets. He uses the excess cash for sales in order to acquire assets through the repo market and earns a return r_R.

Figure 6
Liquidity Constraint: Flow Chart

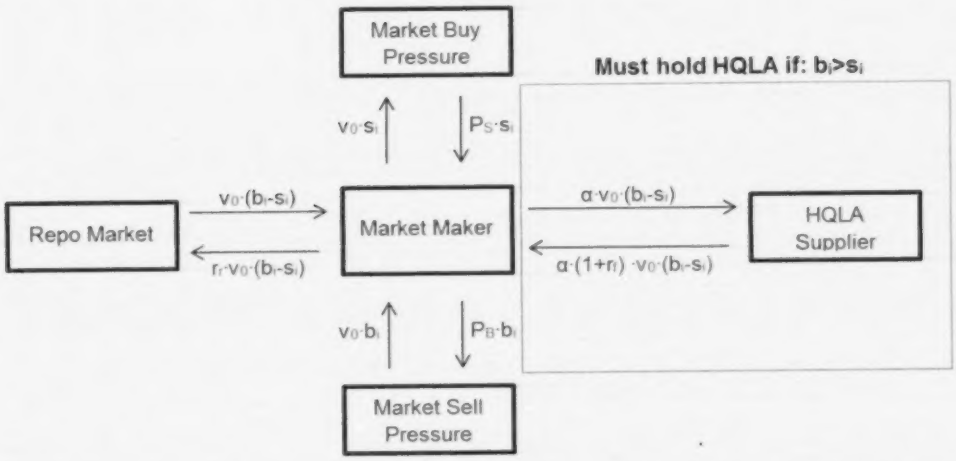
Market Buy Pressure
Must hold HQLA if: $b_i > s_i$
$v_0 \cdot s_i$
$P_S \cdot s_i$
Repo Market
$v_0 \cdot (b_i - s_i)$
$r_f \cdot v_0 \cdot (b_i - s_i)$
Market Maker
$\alpha \cdot v_0 \cdot (b_i - s_i)$
$\alpha \cdot (1+r_f) \cdot v_0 \cdot (b_i - s_i)$
HQLA Supplier
$v_0 \cdot b_i$
$P_B \cdot b_i$
Market Sell Pressure

Figure 6 illustrates the model with a liquidity constraint. A group of N market makers provide liquidity by both buying and selling the asset as Cournot competitors. Market makers access the repo market in order to acquire either more assets or cash. If a market maker wishes to buy more than he sells, he requires cash to do so. He uses this excess inventory for collateral and acquires the cash through the repo market at cost r_R. If a market maker wishes to sell more than he buys, he requires additional assets. He uses the excess cash for sales in order to acquire assets through the repo market and earns a return r_R. The liquidity constraint forces the market maker to hold HQLA if he uses to repo market in order to acquire cash. For every unit of cash he acquires through repo, he must hold α units of the HQLA. This HQLA must be financed through the market maker's fixed assets, costing r_A and paying a return of r_F.

Figure 7
Leverage Ratio: Flow Chart

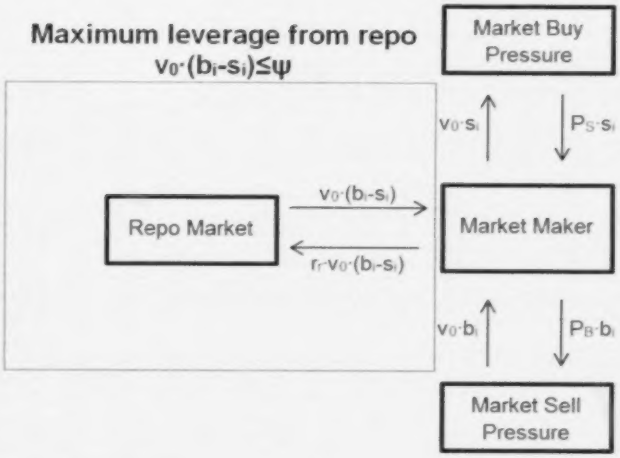

Figure 7 illustrates the model with a leverage ratio constraint. A group of N market makers provide liquidity by both buying and selling the asset as Cournot competitors. Market makers access the repo market in order to acquire either more assets or cash. If a market maker wishes to buy more than he sells, he requires cash to do so. He uses this excess inventory for collateral and acquires the cash through the repo market at cost r_B. If a market maker wishes to sell more than he buys, he requires additional assets. He uses the excess cash for sales in order to acquire assets through the repo market and earns a return r_R. The leverage ratio constraint puts a limit on the amount of debt a market maker is able to acquire during trading. Given his underlying capital structure, a market maker is able to acquire a maximum quantity ψ of debt from the repo market. This creates a limit on the amount of cash the market maker is able to acquire by repo, such that $v_0(b_i - s_i) \leq \psi$.

Figure 8
Liquidity and Leverage Ratio Constraints: Flow Chart

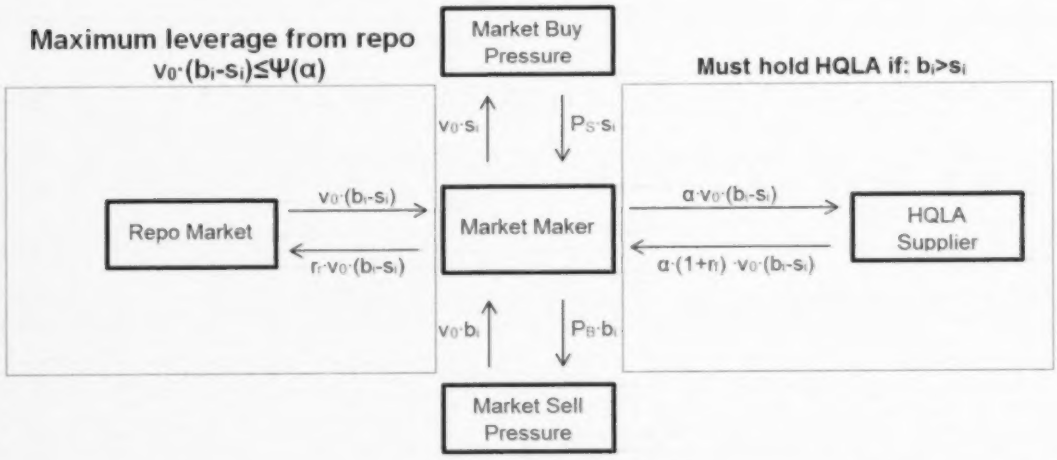

Figure 8 illustrates the model with liquidity and leverage ratio constraints. A group of N market makers provide liquidity by both buying and selling the asset as Cournot competitors. Market makers access the repo market in order to acquire either more assets or cash. If a market maker wishes to buy more than he sells, he requires cash to do so. He uses this excess inventory for collateral and acquires the cash through the repo market at cost r_R. If a market maker wishes to sell more than he buys, he requires additional assets. He uses the excess cash for sales in order to acquire assets through the repo market and earns a return r_R. The liquidity constraint forces the market maker to hold HQLA if he uses the repo market in order to acquire cash. For every unit of cash he acquires through repo, he must hold α units of the HQLA. This HQLA must be financed through the market maker's fixed assets, costing r_A and paying a return of r_F. The leverage ratio constraint puts a limit on the amount of debt a market maker is able to acquire during trading. With both the liquidity and leverage ratio constraints, the market maker acquires debt from both the repo market and the market for HQLA. Given his underlying capital structure, a market maker is able to acquire a maximum quantity Ψ of debt from both markets. This creates a limit on the amount of cash the market maker is able to acquire by repo, such that $v_0(b_i - s_i) \leq \Psi$.